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EXECUTIVE SUMMARY

THE USE OF DEFORMATION PLASTICITY THEORY VERSUS INCREMENTAL PLASTICITY IN EPFM ANALYSIS

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In the Darlington Leak-Before-Break program, Task Group B7 is involved in the calculation of the J-Integral from elastic-plastic finite element results. The use of either the incremental plasticity or deformation plasticity theory stress/strain constitutive relations in these calculations is discussed in this report. Theoretical requirements for the calculation of the J-Integral are given. Inherent differences between the two constitutive models are illustrated through a simple test case. Recommendations are given to check the validity of the J-Integral calculation. Also given are the relative advantages and disadvantages of using the deformation plasticity model rather than incremental plasticity.

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In the Darlington Leak-Before-Break program, Task Group B7 is involved in the calculation of the J-Integral from elastic-plastic finite element results. The use of either the incremental plasticity or deformation plasticity theory stress/strain constitutive relations in these calculations is discussed in this report. Theoretical requirements for the calculation of the J-Integral are given. Inherent differences between the two constitutive models are illustrated through a simple test case. Recommendations are given to check the validity of the J-Integral calculation. Also given are the relative advantages and disadvantages of using the deformation plasticity model rather than incremental plasticity.

1.0 INTRODUCTION

As a part of the Darlington Leak-Before-Break (LBB) effort, Task Group B7 is involved in the application of Elastic-Plastic Fracture Mechanics (EPFM) to crack analyses. The methodology is based on the calculation of the J-Integral/1/ for a loaded crack and comparing with the material's J-Integral fracture resistance as a fracture criterion. These analyses are being performed by using the finite-element computer code ABAQUS/2,3/. Currently both the incremental plasticity and deformation plasticity theory stress/strain relations are available in the code. The need to identify the relative merits of using either constitutive model prompted this investigation.

Some theoretical requirements for applying the J-Integral to elastic-plastic problems will be briefly outlined. The results of a simple numerical test case comparing the two constitutive models will be presented. Some recommendations to check the

validity of the calculation of the J-Integral in conjunction with the incremental plasticity model will be given. The relative advantages and disadvantages of using the deformation plasticity model in place of the incremental plasticity model will then be summarized.

2.0 REQUIRED STRESS/STRAIN RELATION

In reference/1/ the path independence of the J-Integral is demonstrated for a crack in a linear or nonlinear elastic two-dimensional planar solid. For this case the J-Integral is written as

$$J = \int_{\Gamma} \left(W \, dy - T_i \frac{\partial u_i}{\partial x} \, ds \right) \tag{2.1}$$

where W is the strain energy density, x and y are spatial coordinates colinear and perpendicular to the crack, respectively, T_i are tractions, u_i are displacements and ds is an incremental length along the contra-clockwise contour Γ (See Figure 1). The J-Integral has a physical meaning as the energy flux to the crack tip per unit crack extension.

The first term in equation (2.1) was obtained by an integration by parts:

$$\int_{\Gamma} W \, dy = \int_{A} \frac{\partial W}{\partial x} \, dA \tag{2.2}$$

where A is the area enclosed by Γ. Furthermore, the right-hand side of equation (2.2) was arrived at through the relation

$$\frac{\partial W}{\partial x} = \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x} \tag{2.3}$$

where σ_{ij} is the stress tensor and ε_{ij} is the strain tensor. The strain energy density is defined as

$$W = \int_{0}^{\varepsilon_{ij}} \sigma_{ij} \, d\varepsilon_{ij} \, . \tag{2.4}$$

By assuming the strain energy density to be an explicit function of strains only one can write

$$\frac{\partial W}{\partial x} = \frac{\partial W}{\partial \varepsilon_{ij}} \frac{\partial \varepsilon_{ij}}{\partial x} \, . \tag{2.5}$$

From equation (2.4),

$$\frac{\partial W}{\partial \varepsilon_{ij}} = \sigma_{ij} \tag{2.6}$$

and equation (2.3) would be satisfied.

From equation (2.4) it is clear that W will be an explicit function of only strain if σ_{ij} is also a unique function of strain. Thus one needs a stress/strain relation of the form

$$\sigma_{ij} = f_{ij}(\varepsilon_{xx}, \varepsilon_{yy}, \ldots) \quad . \tag{2.7}$$

The total strain is written as the sum of the elastic strain ε^e_{ij} and the plastic strain ε^p_{ij}.

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij} \tag{2.8}$$

A unique relation exists between stress and elastic strain. What remains is to define a unique relation between stress and total plastic strain. Such a relation will be developed in the next section for deformation plasticity theory.

3.0 DEFORMATION PLASTICITY THEORY

The Prandtl-Reuss stress/incremental plastic strain relation is/4/

$$d\varepsilon^p_{ij} = \frac{3}{2} \frac{S_{ij}}{\sigma_e} \, d\varepsilon_p \tag{3.1}$$

where $d\varepsilon^p_{ij}$ is the plastic strain increment tensor, $d\varepsilon_p$ is the effective plastic strain increment, S_{ij} is the deviatoric stress tensor and σ_e is the equivalent stress. Included in the incremental plasticity model is the yield criterion/2/,

$$\sigma_e = \sigma_{ys} \quad , \tag{3.2}$$

where σ_{ys} is the yield stress in uniaxial tension. Plastic straining will result when σ_e reaches σ_{ys}. For a work-hardening material σ_{ys} will also rise to match σ_e. If σ_e decreases, no plastic straining will occur until σ_e once again exceeds the history-dependent value of σ_{ys}.

By restricting the loading at each point in the solid to be proportional the stress components will also increase in a proportional fashion. In this case

$$\sigma_{ij} = Q \, \sigma_{ij_o} \quad , \tag{3.3}$$

$$S_{ij} = Q \, S_{ij_o} \quad , \tag{3.4}$$

$$\sigma_e = Q\,\sigma_{e_o} \quad , \tag{3.5}$$

where Q is a scalar function (of position but not of stress or strain) and the subscript "o" denotes some reference stress state. Upon substituting equations (3.4) and (3.5) into (3.1) one has

$$d\varepsilon^p_{ij} = \frac{3}{2}\,\frac{S_{ij_o}}{\sigma_{e_o}}\,d\varepsilon_p \quad . \tag{3.6}$$

Equation (3.6) can be integrated immediately to yield a general constitutive relation for deformation plasticity,

$$\varepsilon^p_{ij} = \frac{3}{2}\,\frac{\varepsilon_p}{\sigma_e}\,S_{ij} \quad . \tag{3.7}$$

This relation has the same form as equation (2.7), with ε_p considered as a scaling function. The deformation plasticity model has no yield condition. Thus equation (3.7) is equivalent to a nonlinear elastic constitutive relation.

The constitutive equation used in the deformation plasticity model in ABAQUS is/5/

$$\varepsilon^p_{ij} = \frac{3\alpha}{2E\sigma_o^{\,n-1}}\,\sigma_e^{\,n-1}\,S_{ij} \tag{3.8}$$

where E is Young's modulus, and α, n are material constants in the Ramberg-Osgood idealization of the true stress/true strain curve.

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\sigma}{\sigma_o} + \alpha\left(\frac{\sigma}{\sigma_o}\right)^n \tag{3.9}$$

In equation (3.8) σ_e^{n-1} is considered to be a scaling function.

Although the deformation plasticity model should provide a path-independent J-Integral, it will be demonstrated that the field solution generated by this material model will be inaccurate in cases involving significantly nonproportional loading.

4.0 A SIMPLE COMPARISON OF DEFORMATION PLASTICITY MODEL AGAINST INCREMENTAL PLASTICITY

A test case was performed involving a thin plate subjected to bi-axial membrane loads. The model is summarized in Figure 2. Uniform surface tractions t_x and t_y were applied to the two free sides. The load history is divided into three stages, as

illustrated in Figure 3. The first stage of loading was proportional, the second stage was nonproportional, while the third stage was proportional. The case was modelled by using the "benchmark" incremental plasticity and then the deformation plasticity models outlined in Section 3. The case was modelled numerically by running the computer code ABAQUS. A single 8-noded shell element was used. The element was loaded well into the plastic range in four equal increments per loading stage. As a check, the case was also modelled analytically by using the relations given in Section 3. Results were essentially identical to those obtained from the ABAQUS runs.

In Figure 4 the normal stress components σ_{xx} and σ_{yy} from the incremental and deformation plasticity solutions are compared. As expected the models produced essentially identical stresses in response to the distinctly nonproportional loads in the second stage. The iterations for the solution diverged in the deformation plasticity run beyond the normalized time of 1.0. Stresses and strains were uniform through the element. The nonproportional loading is illustrated by the plot of the history of σ_{yy}/σ_{xx} in Figure 5. For proportional loading all the points would lie on a horizontal line.

In Figure 6 are compared the plastic strain components ε^p_{xx} and ε^p_{yy} from the two solutions. The strains are essentially identical during first stage, during which the applied load was still proportional. The effects of Poisson's ratio = 0.5 for incompressible plasticity is evident in the behaviour of ε^p_{yy}.

During the second stage of loading the plastic strains from the two solutions differ markedly in that the plastic strains from the incremental plasticity solution remained constant. The history of the equivalent stress σ_e is plotted in Figure 7. Since in stages 2 and 3 σ_e did not exceed the value of σ_e, and hence σ_{ys}, at the end of stage 1, no further plastic straining occurred. In the deformation plasticity model the total plastic strains are uniquely related to the total deviatoric stresses through equation (3.8). The model has no yield criterion and does not include the past history of any plastic deformation. The deviatoric stress histories are plotted in Figure 8. At the end of stage 2 the deviatoric stress components were equal and therefore the plastic strain components from the deformation plasticity run also became equal. Thus the deformation plasticity model behaved as a nonlinear elastic model, and produced plastic strains much different from the "benchmark" incremental plasticity solution.

5.0 DISCUSSION

5.1 Validity of J-Integral Calculations

The elastic-plastic J-Integral is calculated in the computer code ABAQUS by using the virtual crack extension method by Parks/6/. The method is equivalent to the more formal contour/area integrations used to obtain J, given by equation (2.1) for the planar case.

In Section 2 it was shown that one requirement for path independence of the elastic-plastic J-Integral is that total stresses be uniquely related to total strains. Otherwise the calculated quantity would not be the energy flux to the crack tip per unit crack extension. In Section 3 it was shown that a unique total stress/total strain relation exists for the deformation plasticity model. Path independence of the J-Integral would be ensured by the application of this model. The deformation plasticity model will produce "exact" results when the stress components increase in a proportional manner, such as under proportional applied loads.

However, it was demonstrated in Section 4 that when the loading is nonproportional, the deformation plasticity model will produce a solution which is incorrect relative to that produced by the "benchmark" incremental plasticity model. The deformation plasticity model should still produce a path-independent J-Integral, but for the wrong field solution. Clearly, the application of the J-Integral methodology in such a case would be invalid.

Many problems are expected to involve nearly proportional increases in the stress components. This would be expected in the region surrounding the crack tip, which is dominated by the Hutchinson-Rice-Rosengren (HRR) singlular stress/deformation field/8,9/. In these cases the incremental plasticity model could be applied to ensure a reasonably accurate field solution. Checks on the calculation of the elastic-plastic J-Integral should be made to determine if equation (2.3) is valid and one is actually calculating the energy flux to the crack tip. Given below are validity tests which have been recommended by external consultants to the LBB program/7/.

a) Calculate the J-Integral for several contours and check for path (contour) independence of the J-Integral values.

b) Calculate the J-Integral by using the alternative definition of J as the energy release rate/1/,

$$J = \frac{-\Delta U}{\Delta a} \quad , \qquad (5.1)$$

where ΔU is the change in the total energy potential of the structure due to a small crack length increment Δa. Equation (5.1) should produce the same result as the virtual crack extension method in ABAQUS.

c) Check that stresses and strains are monotonically increasing.

5.2 Advantages and Disadvantages of Deformation Plasticity

The advantages and disadvantages of using the deformation plasticity model in ABAQUS rather than the incremental plasticity model are:

Advantages

1. The calculated elastic-plastic J-Integral should always be path independent and equal to the energy flux to the crack tip for the given field solution.

2. The use of the Ramberg-Osgood idealized stress/strain curve (see equation 3.9) allows scaling of solutions according to the "Engineering Approach"/10/.

3. Shorter required CPU times have been realized for large problems.

Disadvantages

1. In cases involving nonproportional loading (stressing) the solution produced by the deformation plasticity model will be significantly different than that produced by the "benchmark" incremental plasticity model.

2. The deviation of the fitted Ramberg-Osgood idealized stress/strain curve from the actual curve will lead to a loss in accuracy.

6.0 CONCLUSIONS

If the loads or resultant stresses are not completely proportional, the use of the deformation plasticity model should still ensure path independence of the J-Integral. However, the field solution from the deformation plasticity model will be different from that obtained by using the incremental plasticity model. In these cases the increase in stress components in the HRR singularity-dominated region surrounding the crack tip

may still be nearly proportional. The use of incremental plasticity should produce reasonable estimates of the J-Integral provided the contours are restricted to this region and the validity checks a) to c) mentioned above are satisfied.

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DAS:em

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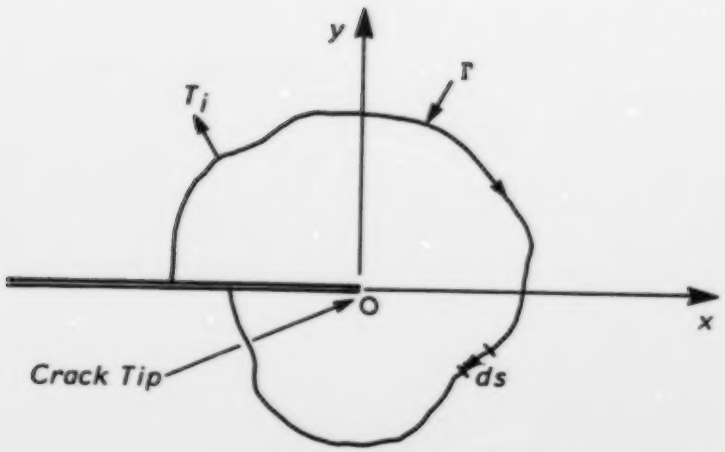
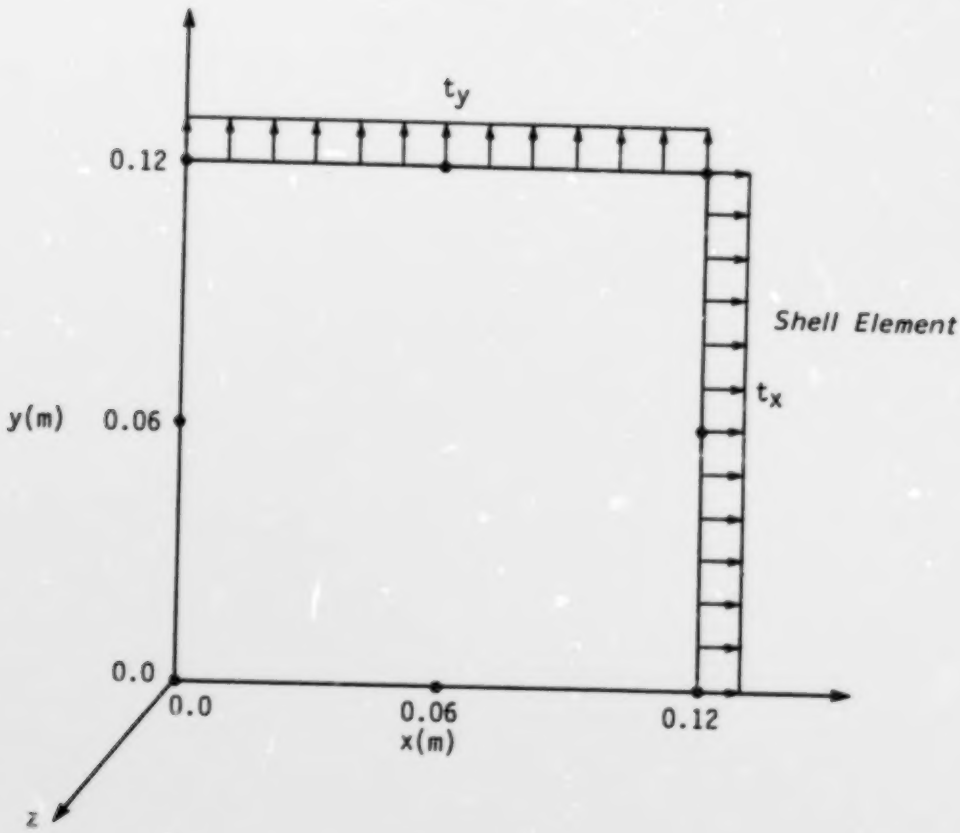


FIGURE 1

ILLUSTRATION OF THE J-INTEGRAL CONTOUR
FOR A TWO-DIMENSIONAL PLANAR PROBLEM



• - *Indicates a Node*

Thickness = 0.040 m

E = 182.8 x 10^3 MPa

ν = 0.3

σ_o = 290.3 MPa

n = 5.2108

α = 2.0394

FIGURE 2
DESCRIPTION OF TEST CASE

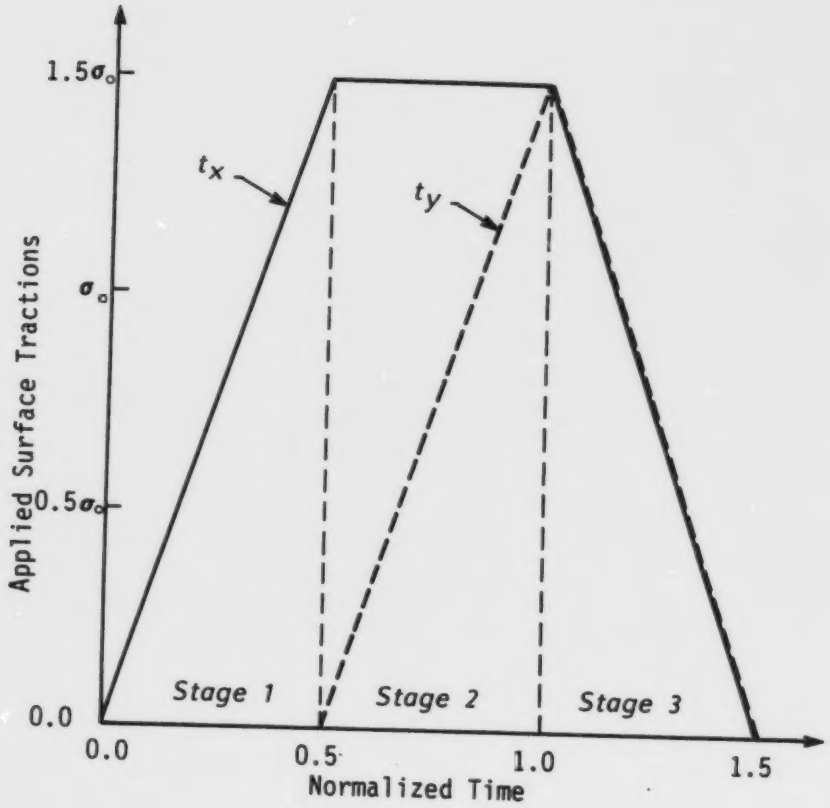


FIGURE 3
APPLIED LOAD HISTORY

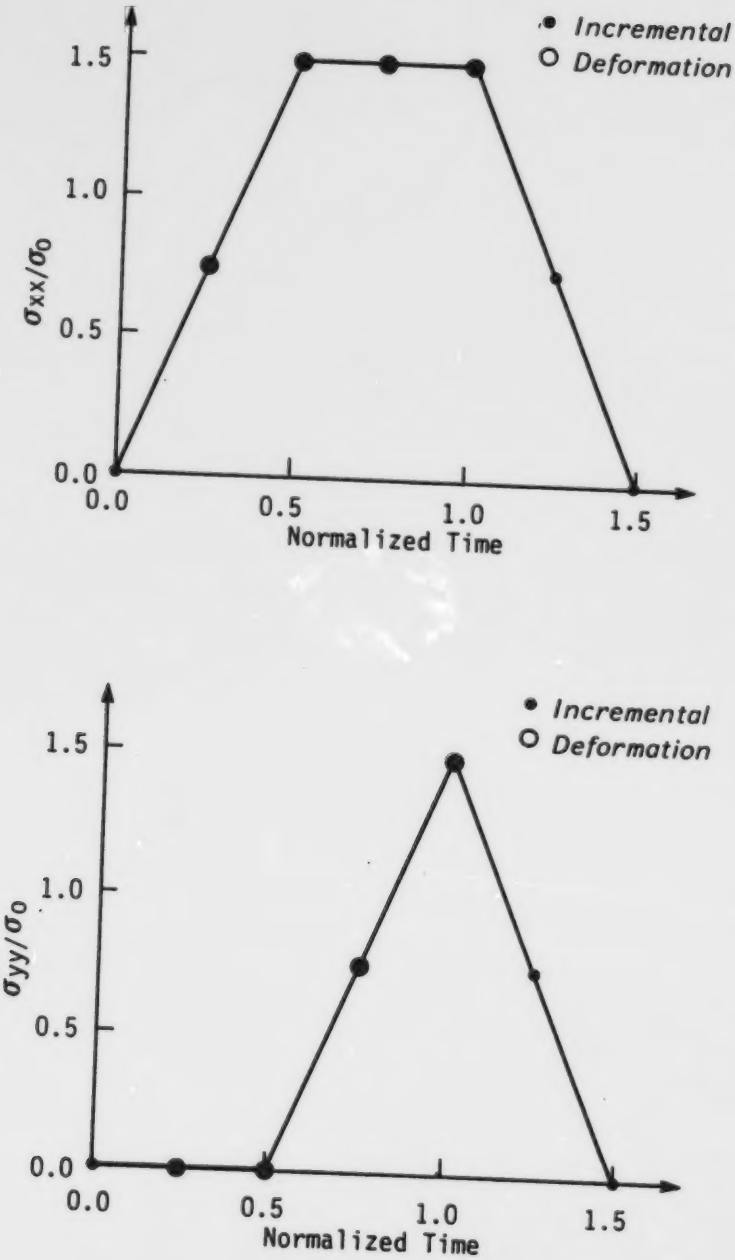


FIGURE 4
COMPARISON OF STRESS COMPONENT HISTORIES

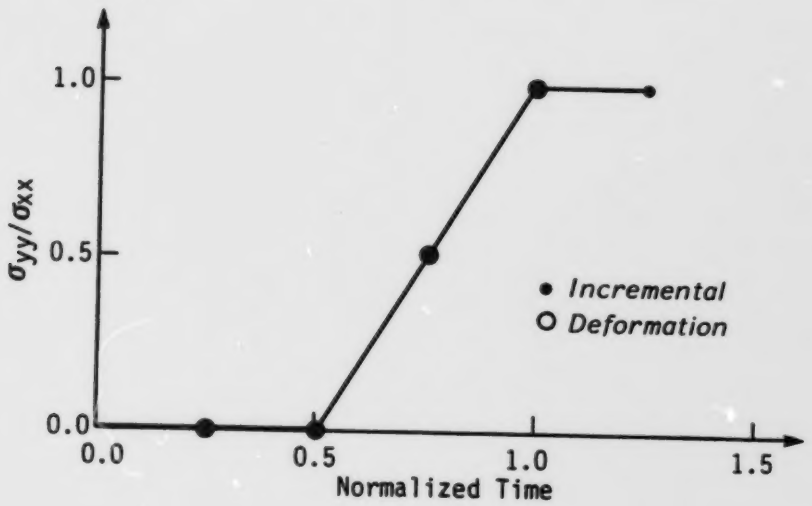


FIGURE 5
COMPARISON OF STRESS RATIO

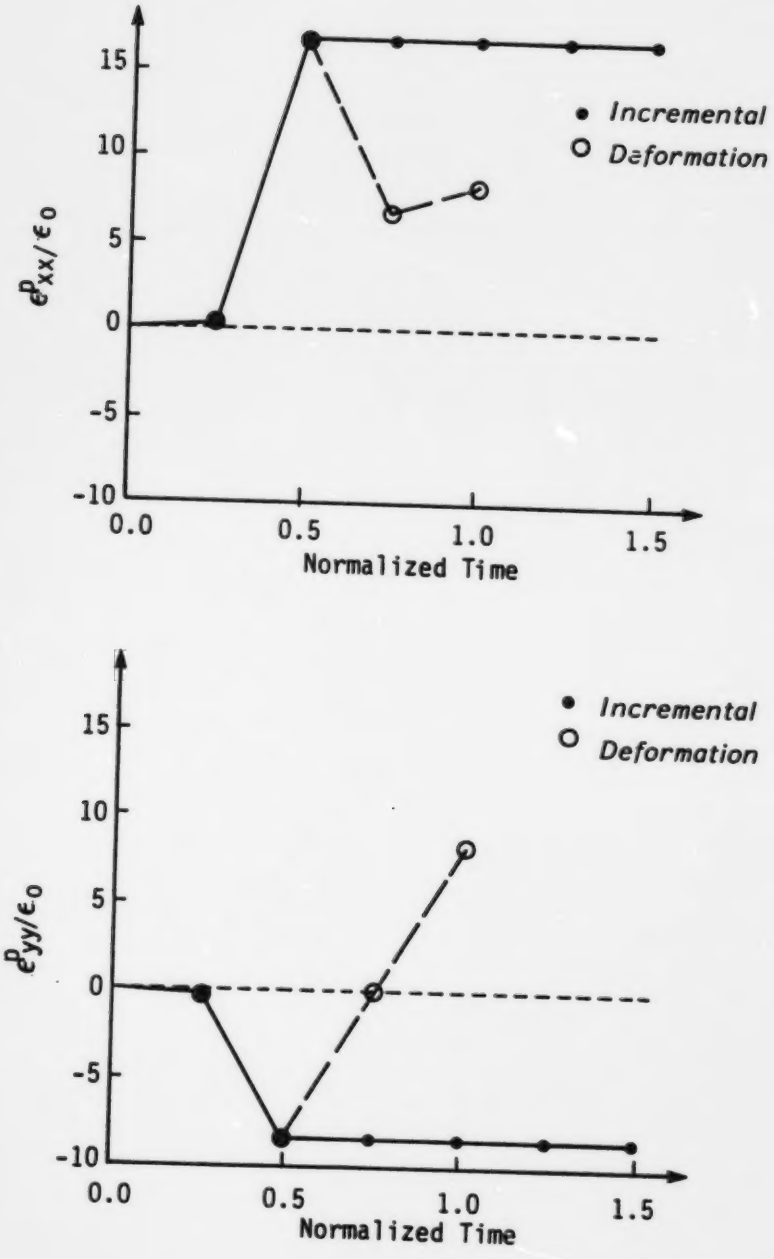


FIGURE 6
COMPARISON OF PLASTIC STRAIN COMPONENT HISTORIES

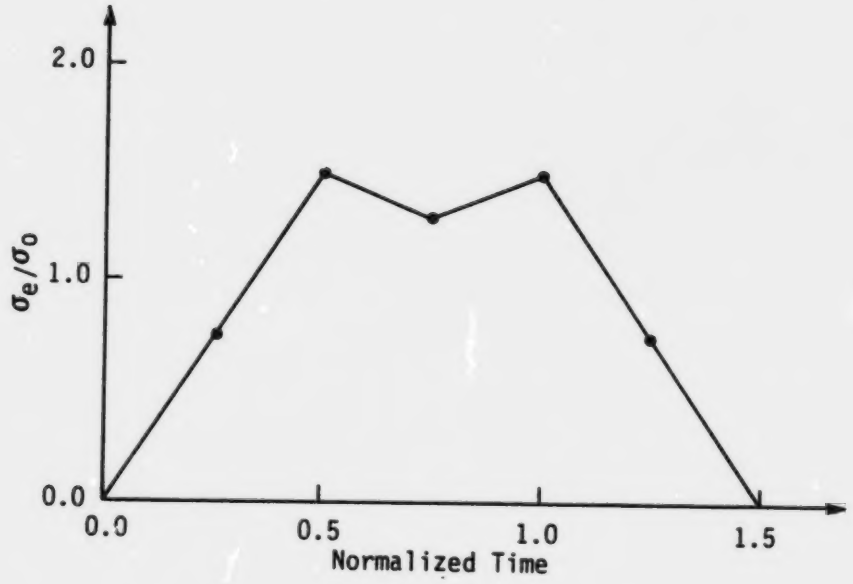


FIGURE 7
HISTORY OF EFFECTIVE STRESS

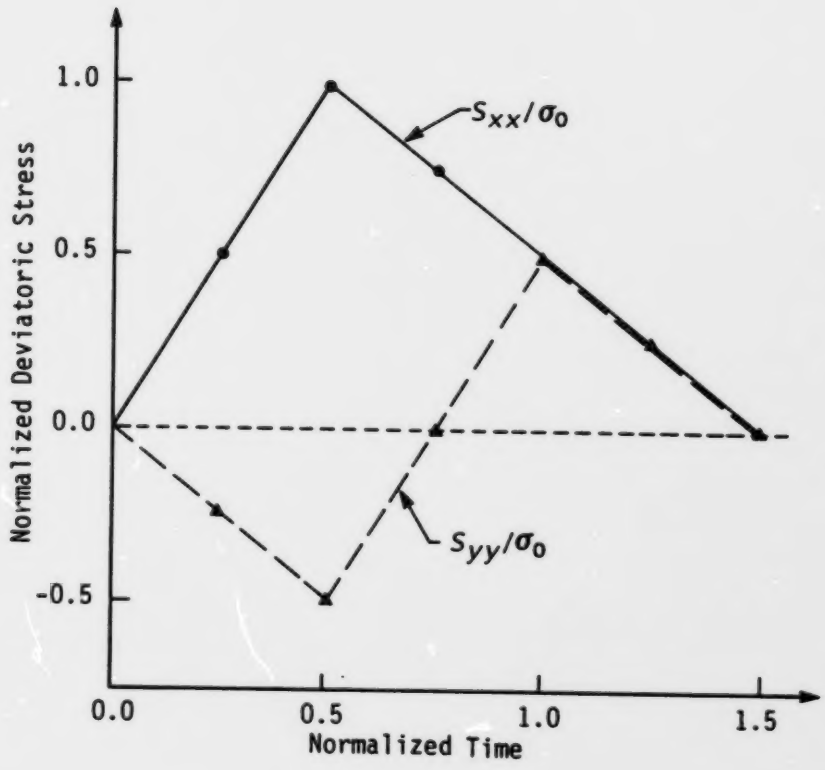


FIGURE 8
HISTORY OF DEVIATORIC STRESS COMPONENTS